Subgroup Meeting #3

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DMRG for the SIS Model

- In ordinary quantum mechanics, the operators for the observables live in $\mathcal{L}(\mathcal{H}, \mathcal{H})$, where $\mathcal{H}$ is a $\mathbb{C}-$**Hilbert space**.
- In the SIS model, the linear operators also live in $\mathcal{L}(\mathcal{H}, \mathcal{H})$, but here $\mathcal{H}$ is a $\mathbb{R}-$**Hilbert space**.
- The linear operators in the SIS model are **not hermitian**.
- Why does the DMRG work?

  Is the Hamiltonian diagonalizable?
  What is the spectrum of the Hamiltonian?
  How to get the NESS from the Hamiltonian?

DMRG for the SIS Model

- Diagonalizability:

  Master equation: $\partial_t |P(t)\rangle = \hat{W} |P(t)\rangle$

  The state: $|P(t)\rangle = e^{t\hat{W}} |P(0)\rangle$

  Write $\hat{W}$ in its Jordan form

  $\rightarrow$ $\hat{W}$ is **diagonalizable** or the state will diverge.

- The spectrum:

  $\hat{W}$ is diagonalizable

  $\rightarrow$ The eigenvalues are **negative** or the state will diverge.

- NESS:

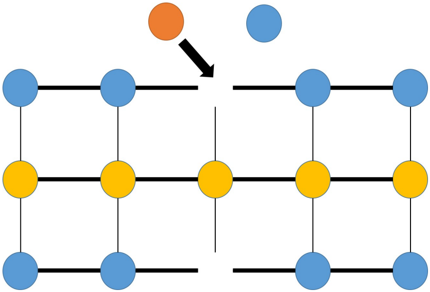
  Entries of $\hat{W}$ are $e^{\lambda_i t}$ goes to zero as $t \rightarrow \infty$ if $\lambda_i < 0$

  $\rightarrow$ the only important entry is $e^{0t} = 1$

  $\rightarrow$ the NESS is the **eigenvector of** $\hat{W}$ **with** $\lambda = 0$

DMRG for the SIS Model

- Variational problem: $\min_A (A^\dagger \hat{W}_{eff} A - \lambda A^\dagger \hat{N} A)$
  $\rightarrow$ Consider the gradient: $\nabla_{A^\dagger}(A^\dagger \hat{W}_{eff} A - \lambda A^\dagger \hat{N} A) = 0$
  $\rightarrow$ Generalized eigenvalue problem: $\hat{W}_{eff} A = \lambda \hat{N} A$



From NRG to DMRG

- Consider a one-dimensional quantum chain:

  Hilbert space: $\mathcal{H} = \bigotimes_{i=1}^{N} \mathcal{H}_i$

  Dimension: $\dim \mathcal{H} = \prod_{i=1}^{N} \dim \mathcal{H}_i$, growing rapidly as N growing.
- The Numerical Renormalization Group (NRG) is a kind of numerical algorithm that can find the **groundstate** of a system.
- The key idea of NRG:
  Truncation $\rightarrow$ **Add a new site** $\rightarrow$ **Diagonalization** $\rightarrow$ **Truncation** $\rightarrow \cdots \rightarrow$ **Until Convergence**

From NRG to DMRG

- NRG works well in some impurity models but fails in **strongly correlated systems**. **Whole**$=\sum$ **Parts**$+$**?**.
- The **entanglement effect**, related to many interesting phenomena, is not considered in NRG.
- In the density matrix renormalization group (DMRG), we use the **density matrix** to measure the **entanglement entropy**:

From NRG to DMRG

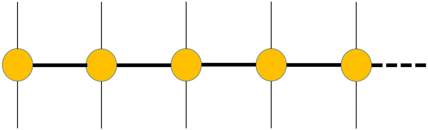
- Algorithms

    Infinite-size DMRG
    Finite-size DMRG

`https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html`

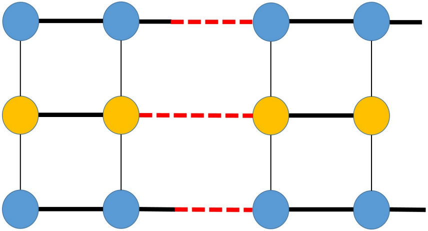
DMRG as Renormalization Group

- We can obtain the ground state by diagonalizing the infinite tensor:



This is impossible.

- We can consider the truncation to reduce the dimension of the MPO:



Variational Perspective of Finite-Size DMRG

- We want to find the ground state of a quantum state:

  Ground state energy: $\inf_{|\Psi\rangle} \frac{\langle\Psi|\hat{H}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$.

  Optimization problem: $\min_{|\Psi\rangle}(\langle\Psi|\hat{H}|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle)$.
- The quantum state $|\Psi\rangle$ can be rewritten in **MPS**.
- In principle, we can solve it by implementing the variational method concerning **all the tensors in MPS**
  $\rightarrow$ **impossible in the computers** $\rightarrow$ **implement tensor by tensor**.
- Solve the variational problem
  $\min_A(\langle\Psi|\hat{H}|\Psi\rangle - \lambda\langle\Psi|\Psi\rangle) = \min_A(A^\dagger \hat{H}_{eff} A - \lambda A^\dagger \hat{N} A)$

  A: variational parameter

  $\hat{H}_{eff}$: effective Hamiltonian, $\langle\Psi|\hat{H}|\Psi\rangle$ without $A, A^\dagger$.

  $\hat{N}$: normalization matrix, $\langle\Psi|\Psi\rangle$ without $A, A^\dagger$.

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    Conjugate gradient method
    Gradient Descendent
    Tangent space method