Group Meeting #4

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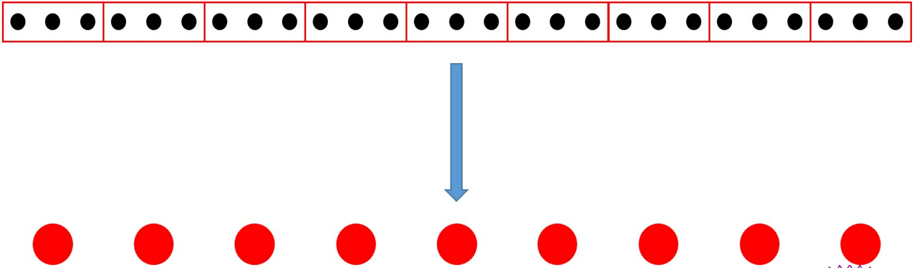
Renormalization Group and Matrix Product State

- The density matrix renormalization group (DMRG) algorithm can be implemented in two languages:

  Physical: Renormalization group (RG)
  Mathematical: Matrix product state (MPS)

Introduction to RG

- Idea of RG: **killing degree of freedom (maybe infinite)**



Algorithms (RG)

- Algorithms:
  https://kikiyenhaoyang.github.io/kikiyen/Web/TN.html

Variational Perspective of Finite-Size DMRG in MPS

- We want to find the ground state of a quantum state:

  Ground state energy: $\inf_{|\Psi\rangle} \frac{\langle\Psi|\hat{H}|\Psi\rangle}{\langle\Psi|\Psi\rangle}$.

  Optimization problem: $\min_{|\Psi\rangle}(\langle\Psi|\hat{H}|\Psi\rangle - \lambda(\langle\Psi|\Psi\rangle - 1))$.
- The quantum state $|\Psi\rangle$ can be rewritten in **MPS**.
- In principle, we can solve it by implementing the variational method concerning **all the tensors in MPS**
  $\rightarrow$ **impossible in the computers** $\rightarrow$ **implement tensor by tensor**.
- Solve the variational problem
  $\min_A(\langle\Psi|\hat{H}|\Psi\rangle - \lambda(\langle\Psi|\Psi\rangle - 1)) = \min_A(A^\dagger \hat{H}_{eff} A - \lambda(A^\dagger \hat{N} A - 1))$

  A: variational parameter

  $\hat{H}_{eff}$: effective Hamiltonian, $\langle\Psi|\hat{H}|\Psi\rangle$ without $A, A^\dagger$.

  $\hat{N}$: normalization matrix, $\langle\Psi|\Psi\rangle$ without $A, A^\dagger$.

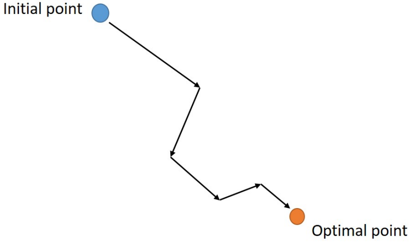
Variational Perspective of Finite-Size DMRG in MPS

- Variational problem: $\min_A (A^\dagger \hat{H}_{eff} A - \lambda(A^\dagger \hat{N} A - 1))$
  $\rightarrow$ Consider the gradient: $\nabla_{A^\dagger}(A^\dagger \hat{H}_{eff} A - \lambda(A^\dagger \hat{N} A - 1)) = 0$
  $\rightarrow$ Generalized eigenvalue problem: $\hat{H}_{eff} A = \lambda \hat{N} A$

  Conjugate gradient method (CG)

  Gradient Descendent

  Tangent space method



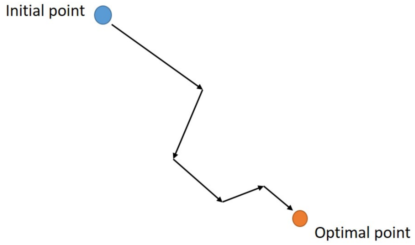
Variational Perspective of Finite-Size DMRG in MPS

- Solving the linear systems $Ax = b$
  Gaussian elimination $\rightarrow$ **complexity:** $\mathcal{O}(n^3)$
  $\rightarrow$ For a huge linear system, it may be incredibly expensive.
  CG: $\rightarrow$ **complexity:** $\mathcal{O}(m\sqrt{k})$

  m : number of nonzero entries

  k : condition number

  Key idea: Iteratively find the locally optimal solution.



Variational Perspective of Finite-Size DMRG in MPS

- Variational problem: $\min_A (A^\dagger \hat{H}_{eff} A - \lambda(A^\dagger \hat{N} A - 1))$
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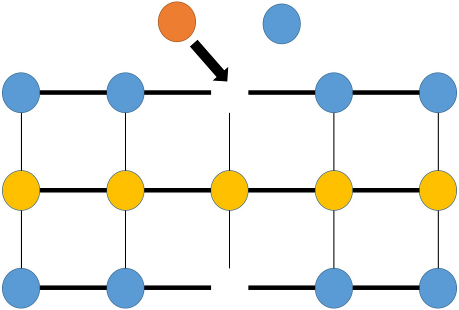

Algorithms (MPS)

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